Assignment II: Singular homology

March 27, 2023

Practice problems

1. Try the following problems from Section 2.1 (Pages 132-133) of Hatcher: 11, 14, 16, 17, 20, 21, 27, 28, and 29.

2. Prove that the boundary map in the long exact sequence of a pair is a homomorphism (at all levels).

3. Verify that the induced homomorphism f_*, check that the map is well-defined at all levels (fill in the gaps).

4. Consider the proof of theorem that asserts that homotopic maps f, g induce the same homomorphism on homology.

   (a) Describe explicitly, the do the subdivision of the prism (i.e. the standard n-simplex $\times I$) into $(n+1)$-simplices for the case $n = 2$.

   (b) For the prism operator P, show that $\partial P + P\partial = g_\# - f_\#$.

5. Describe explicitly the retraction $D^n \to S^{n-1}$ in the proof of Brouwer's fixed point theorem.

6. Verify that here exists a long exact sequence for reduced homology groups analogous to the long exact sequence of homology groups. Moreover, establish that both reduced homology groups are homotopic and the relative homology groups of the pair (X, A) are isomorphic when $A \neq \emptyset$.

7. Give examples of maps f, g such that $f_* = g_*$, but f is not homotopic to g.

8. Does there exist CW-complex structures on punctured surfaces or surfaces with boundaries?

9. Does every Δ-complex also admit a CW-complex structure?

10. Prove that a homeomorphisms of a surface take boundary to boundary.

11. In the proof (of the infinite-dimensional case) of the fact that $H_n(X, A) \cong H_n^\Delta(X, A)$ from class, why do the sets $U_i = X - \cup_{j \neq i}\{x_j\}$ from an open cover with no finite subcover?

12. For a $f : X \to Y$, show that there exists a well-defined suspension map $Sf : SX \to SY$.

13. If maps $f, g : X \to Y$ are homotopic, then are their suspensions Sf, Sg homotopic?

14. Can every map on S^{n+1} be realized as the suspension of a map on S^n? In this regard, verify whether the following statements are true or false.

    (a) A rotation of S^2 is the suspension of a rotation of S^1.

    (b) The antipodal map on S^{n+1} is the suspension of the antipodal map on S^n.

Problems for submission

Please turn in your solutions to the following problems from the practice problems listed above by the due date. (Due 26/2/2023)

- Problem 1: Exercises 17, 20, 21, and 29
- Problems 6 and 9